LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **JULY 2024**

UST 3502 – MATRIX AND LINEAR ALGEBRA

Date: 13-06-2024 Dept. No. Max. : 100 Marks

Time: 10:00 AM - 01:00 PM

	SECTION A - K1 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
1.	**Define the following**
a)	Triangular matrix.
b)	Linear Span.
c)	Orthogonal Matrix.
d)	Eigen roots and Eigen vectors.
e)	Index and signature of the matrix.
2.	**Fill in the blanks**
a)	A matrix A such that $A^2 = A$ is called______.
b)	If two rows (or two columns) of a matrix are identical, the value of the determinate is ___________
c)	Any infinite set is linearly independent if it's every finite subset is______.
d)	The Eigen value of A is 3, -4 and 0 . Then, the Eigen Value of A^2 is ____________.
e)	A real symmetric matrix is positive definite if and only if all its eigen values are ________.
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**True or False**
a)	The Addition of matrices is not always commutative.
b)	The interchange of a pair of columns does not change the rank of a matrix.
c)	If all the elements of a row (or a column) of a matrix are zero, the value of the determinant is non zero.
d)	The characteristic vectors corresponding to distinct characteristic roots of a matrix are linearly independent.
e)	The number of negative square terms in the Q.F is called the Index of Q.F.
4.	**Answer the following**
a)	Write the association between rank of a matrix and its linear independence.
b)	Write the importance of similar matrix in finding the Eigen values.
c)	What is meant by Basis?
d)	Write the Matrix form for the following quadratic form. $Q(x_1, x_2, x_3) = x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_1x_3$.
e)	Write any two properties of Eigen values and Eigen vectors.
	SECTION B - K3 (CO2)
	Answer any TWO of the following **(2 x 10 = 20)**

5.	(i) Prove that A= $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$ $and\ B = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$. are inverse of each other under multiplication. (ii) If A= $\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ then find A^2-5A-14I and hence obtain A^3. (5+5)
6.	$If\ A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ $and\ B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $show\ that$ (A+B)(A-B)≠A^2-B^2.
7.	Examine, if A and B be matrices such that AB = BA, then show that forevery positive integer n (i) $AB^n = B^n A$, (ii) $(AB)^n = A^n B^n$
8.	Determine the characteristic roots and the corresponding characteristic vectors of the matrix. $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 10 = 20)**
9.	Explain the properties of addition of matrices.
10.	Suppose T is a linear transformation, T: $R^2 \rightarrow R^2$ and T $(1,1)^T = (1,2)^T$, $T(0, -1)^T = (3,2)^T$. Find the matrix A of T such that T($\vec{x}$)= A$\vec{x}$ for all $\vec{x}$.
11.	Determine a non-singular matrix P such that $P'AP$ is a diagonal matrix, where, A= $\begin{bmatrix} 7 & 0 & -3 \\ -9 & -2 & 3 \\ 18 & 0 & -8 \end{bmatrix}$. Interpret the result in terms of quadratic forms.
12.	(i) Describe Cramer's rule of finding solutions of simultaneous equations in four unknowns. (ii) Check whether the system of equations x+2y-z=3 3x-y+2z=1, 2x-2y+3z=2, x-y+z=-1 are consistent and solve them.
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 20 = 20)**
13.	(i) Prove that the following matrix satisfies Cayley-Hamilton theorem. $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ (II) Compute A^{-1} for the matrix A= $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ And hence solve the following system of equations -x+2y+5z=2 2x-3y+z=15 -x+y+z=-3 (10+10)
14.	(i) Prove that if A be any n-rowed square matrix, then (Adj A) A = A (Adj A) = AI_n , where I_n is the n-rowed unit matrix.

	(ii) Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$ (10+10)

SECTION E – K6 (CO5)

	Answer any ONE of the following **(1 x 20 = 20)**
15.	(i) Find the number of values of t for which the system of equations $(a-t)x + by + cz = 0$ $bx + (c-t)y + az = 0$ $cx + ay + (b-t)z = 0$ has non-trivial solution. (ii) Find all the Eigenvalues and Eigenvectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. (10+10)
16.	(i) $v_1=(1,3,-1,0)^T$, $v_2=(2,9,-1,3)^T$, $v_3=(4,5,6,11)^T$, $v_4=(1,-1,2,5)^T$, $v_5=(3,-2,6,7)^T$. Is $\{v_1, v_2, v_3, v_4\}$ linear dependent or linearly independent? (ii) Reduce the Quadratic form $Q(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 - 2x_1x_2$ to canonical form through an orthogonal transformation. Find the nature rank, index, signature and also find the non-zero set of values which makes this Quadratic form as zero. (10+10)

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